3-5 Modeling with Nonlinear Regression

For Exercises 1–3, complete each step.

a. Find an exponential function to model the data.

b. Find the value of each model at $x = 20$.

d. The equation is approximately $y = 3.19(1.98)^x$.

e. Use the graph to find the value of $y$ for $x = 20$. 
3-5 Modeling with Nonlinear Regression

2.

**SOLUTION:**

a. Enter the data into the calculator and then select ExpReg.

```
<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>23</td>
</tr>
<tr>
<td>3</td>
<td>124</td>
</tr>
<tr>
<td>4</td>
<td>620</td>
</tr>
<tr>
<td>5</td>
<td>3130</td>
</tr>
<tr>
<td>6</td>
<td>15,600</td>
</tr>
</tbody>
</table>
```

The equation is approximately \( y = 1.04(4.95)^x \).

b. Use the graph to find the value of \( y \) for \( x = 20 \).
3-5 Modeling with Nonlinear Regression

### Table

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>25</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>1.6</td>
</tr>
<tr>
<td>3</td>
<td>0.4</td>
</tr>
<tr>
<td>4</td>
<td>0.09</td>
</tr>
<tr>
<td>5</td>
<td>0.023</td>
</tr>
<tr>
<td>6</td>
<td>0.006</td>
</tr>
</tbody>
</table>

**SOLUTION:**

a. Enter the data into the calculator and then select ExpReg.

![Graphing Calculator](image)

The equation is approximately $y = 24.98(0.25)^x$.

b. Use the graph to find the value of $y$ for $x = 20$.

![Graph](image)
4. **GENETICS** *Drosophila melanogaster*, a species of fruit fly, are a common specimen in genetics labs because they reproduce about every 8.5 days, allowing researchers to study several generations. The table shows the population of drosophila over a period of days.

<table>
<thead>
<tr>
<th>Generation</th>
<th>Drosophila</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80</td>
</tr>
<tr>
<td>2</td>
<td>156</td>
</tr>
<tr>
<td>3</td>
<td>307</td>
</tr>
<tr>
<td>4</td>
<td>593</td>
</tr>
<tr>
<td>5</td>
<td>1180</td>
</tr>
<tr>
<td>6</td>
<td>2314</td>
</tr>
<tr>
<td>7</td>
<td>4612</td>
</tr>
<tr>
<td>8</td>
<td>8943</td>
</tr>
</tbody>
</table>

**SOLUTION:**

a. Find an exponential function to model the data.
b. Use the function to predict the population of drosophila after 93.5 days.

**SOLUTION:**

a. Enter the data into the calculator and then select ExpReg.

The equation is approximately \( y = 40.69(1.96)^x \).

b. The value of \( x \) represents the number of generations. After 93.5 days, there have been \( \frac{93.5}{8.5} \) or 11 generations.
3-5 Modeling with Nonlinear Regression

5. **SHARKS** Sharks have numerous rows of teeth embedded directly into their gums and not connected to their jaws. As a shark loses its teeth, teeth from the next row move forward. The rate of replacement of a row of teeth in days per row increases with the temperature of the water.

<table>
<thead>
<tr>
<th>Temp. (°C)</th>
<th>20</th>
<th>21</th>
<th>22</th>
<th>23</th>
<th>24</th>
<th>25</th>
<th>26</th>
<th>27</th>
</tr>
</thead>
<tbody>
<tr>
<td>Days per Row</td>
<td>66</td>
<td>54</td>
<td>44</td>
<td>35</td>
<td>28</td>
<td>22</td>
<td>18</td>
<td>16</td>
</tr>
</tbody>
</table>

a. Find an exponential function to model the data.

b. Use the function to predict the temperature at which sharks lose a row of teeth in 12 days.

**SOLUTION:**

a. Enter the data into the calculator and then select ExpReg.

![Graph](image)

The equation is approximately $y = 4476(0.81)^x$.

b. Graph the full regression line and locate the intersection of this line and $y = 12$.

![Graph](image)
3-5 Modeling with Nonlinear Regression

6. **WORDS** A word family consists of a base word and all of its derivations. The table shows the percentage of words in an average English text comprised of the most common word families.

<table>
<thead>
<tr>
<th>Word Families</th>
<th>1000</th>
<th>2000</th>
<th>3000</th>
<th>4000</th>
<th>5000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage of Words</td>
<td>73.1</td>
<td>79.7</td>
<td>84.0</td>
<td>86.7</td>
<td>88.6</td>
</tr>
</tbody>
</table>

**a.** Find a logarithmic function to model the data.

**b.** Predict the number of word families that make up 95% of the words in an average English text.

**SOLUTION:**

**a.** Enter the data into the calculator and then select LnReg.

![LnReg Graph](image)

The rounded equation is \( y = 5.84 + 9.74 \ln x \).

**b.** Graph the full regression line and locate the intersection of this line and \( y = 95 \).

![Graph](image)

about 9483 word families
3-5 Modeling with Nonlinear Regression

For Exercises 7–9, complete each step.
a. Find a logarithmic function to model the data.
b. Find the value of each model at \( x = 15 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>42</td>
</tr>
<tr>
<td>3</td>
<td>37</td>
</tr>
<tr>
<td>4</td>
<td>33</td>
</tr>
<tr>
<td>5</td>
<td>31</td>
</tr>
<tr>
<td>6</td>
<td>28</td>
</tr>
<tr>
<td>7</td>
<td>27</td>
</tr>
</tbody>
</table>

7.

**SOLUTION:**

a. Enter the data into the calculator and then select LnReg.

```
LnReg
y = a + b \ln x
a = 50.1158501
b = -12.05620524
```

The equation is approximately \( y = 50.11 - 12.06 \ln x \).

b. Use the graph to find the value of \( y \) for \( x = 15 \).
8. **SOLUTION:**

a. Enter the data into the calculator and then select LnReg.

The equation is approximately \( y = 9.98 - 2 \ln x \).

b. Use the graph to find the value of \( y \) for \( x = 15 \).
9. 

**SOLUTION:**

a. Enter the data into the calculator and then select LnReg.

![Graph of LnReg]

The equation is approximately \( y = 39.95 + 14.44 \ln x \).

b. Use the graph to find the value of \( y \) for \( x = 15 \).

![Graph of LnReg with x=15]

\([0, 20] \text{ scl: 2 by} [0, 100] \text{ scl: 10}\)
10. **CHEMISTRY** A lab received a sample of an isotope of cobalt in 1999. The amount of cobalt in grams remaining per year is shown in the table below.

<table>
<thead>
<tr>
<th>Year</th>
<th>Cobalt (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>877</td>
</tr>
<tr>
<td>2001</td>
<td>769</td>
</tr>
<tr>
<td>2002</td>
<td>674</td>
</tr>
<tr>
<td>2003</td>
<td>591</td>
</tr>
<tr>
<td>2004</td>
<td>518</td>
</tr>
<tr>
<td>2005</td>
<td>454</td>
</tr>
<tr>
<td>2006</td>
<td>398</td>
</tr>
<tr>
<td>2007</td>
<td>349</td>
</tr>
</tbody>
</table>

a. Make a scatter plot of the data.
b. Find a logarithmic function to model the data. Let \( x = 1 \) represent 2000.
c. Predict the amount of cobalt remaining in 2020.

**SOLUTION:**

a. Enter the data into the calculator and then select LnReg.

The equation is approximately \( y = 922.18 - 259.08 \ln x \).

b. Use the graph to find the value of \( y \) for \( x = 21 \).
3-5 Modeling with Nonlinear Regression

For Exercises 11-13, complete each step.
 a. Make a scatter plot of the data.
 b. Find a logistic function to model the data.
 c. Find the value of each model at \( x = 25 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>67</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
</tr>
<tr>
<td>6</td>
<td>89</td>
</tr>
<tr>
<td>8</td>
<td>94</td>
</tr>
<tr>
<td>10</td>
<td>97</td>
</tr>
<tr>
<td>12</td>
<td>98</td>
</tr>
<tr>
<td>14</td>
<td>99</td>
</tr>
</tbody>
</table>

11. **SOLUTION:**

a. Enter the data into the calculator and then select Logistic.

```
Logistic
y=a/(1+Ce^(-bx))
s=.9909218988
b=.3514001119
c=99.64953817
```

The equation is approximately \( y = \frac{99.65}{1 + 0.99e^{-0.35x}} \).

b. Use the graph to find the value of \( y \) for \( x = 25 \).

\([0, 50] \text{ scl: } 5 \ \text{ by } [0, 100] \text{ scl: } 10\)
3-5 Modeling with Nonlinear Regression

12.  

**SOLUTION:**

**a.** Enter the data into the calculator and then select Logistic.

![Logistic regression graph]

The equation is approximately $y = \frac{24.63}{1 + 12.74e^{-0.52x}}$.

**b.** Use the graph to find the value of $y$ for $x = 25$.

![Graph with $x = 25$]

[0, 30] scl: 3 by [0, 30] scl: 3
3-5 Modeling with Nonlinear Regression

13.

**SOLUTION:**

a. Enter the data into the calculator and then select Logistic.

The equation is approximately \( y = \frac{24.63}{1 + 12.74e^{-0.52x}} \).

b. Use the graph to find the value of \( y \) for \( x = 25 \).
3-5 Modeling with Nonlinear Regression

14. **CHEMISTRY** A chemistry is performing a titration in lab. To perform the titration, she uses a burette to add a basic solution of NaOH to a neutral solution. The table shows the pH of the solution as the NaOH is added.

<table>
<thead>
<tr>
<th>NaOH (mL)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7.5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>pH</td>
<td>10</td>
<td>10.4</td>
<td>10.6</td>
<td>11.0</td>
<td>11.3</td>
<td>11.5</td>
<td>11.5</td>
</tr>
</tbody>
</table>

a. Find a logistic function to model the data.
b. Use the model to predict the pH of the solution after 12 milliliters of NaOH have been added.

**SOLUTION:**
a. Enter the data into the calculator and then select Logistic.

```
Logistic
y=c/(1+ae^(-bx))
a=1.648213191
b=3.298179356
c=11.62204751
```

The equation is approximately \( y = \frac{11.62}{1 + 0.165e^{-0.33x}} \).
b. Use the graph to find the value of \( y \) for \( x = 6 \).
For Exercises 1–3, complete each step.

a. Find an exponential function to model the data.

b. Find the value of each model at \( t = 0 \) and \( t = 2 \), and discuss the behavior of each model as \( t \) increases.

c. Discuss the effectiveness of the model to predict the population as time increases significantly beyond the domain of the data.

SOLUTION:

a. Make a scatter plot of the data.

b. Find the value of each model at \( t = 0 \) and \( t = 2 \).

c. Discuss the effectiveness of the model to predict the population as time increases significantly beyond the domain of the data.

The table shows the average length of female king snake eels at various ages.

<table>
<thead>
<tr>
<th>Year</th>
<th>Length (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>12</td>
</tr>
<tr>
<td>2005</td>
<td>16</td>
</tr>
<tr>
<td>2010</td>
<td>18</td>
</tr>
<tr>
<td>2015</td>
<td>20</td>
</tr>
<tr>
<td>2020</td>
<td>22</td>
</tr>
<tr>
<td>2025</td>
<td>24</td>
</tr>
<tr>
<td>2030</td>
<td>26</td>
</tr>
</tbody>
</table>

a. Find a logistic function to model the data.

b. Based on the model, at what population does the 2000 census predict Maine’s growth to level off?

c. Discuss the effectiveness of the model to predict the population as time increases significantly beyond the domain of the data.

SOLUTION:

a. Enter the data into the calculator and then select Logistic.

b. The numerator of the logistic function is 1.43, so, according to the function, Maine’s growth will level off at about 1.43 million.

c. Sample answer: The model only remains effective as long as the birth and death rates remain consistent and the available resources do not change.

3-5 Modeling with Nonlinear Regression

15. CENSUS The table shows the projected population of Maine from the 2000 census. Let \( x \) be the number of years after 2000.

<table>
<thead>
<tr>
<th>Year</th>
<th>Population (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>1.275</td>
</tr>
<tr>
<td>2005</td>
<td>1.319</td>
</tr>
<tr>
<td>2010</td>
<td>1.357</td>
</tr>
<tr>
<td>2015</td>
<td>1.389</td>
</tr>
<tr>
<td>2020</td>
<td>1.409</td>
</tr>
<tr>
<td>2025</td>
<td>1.414</td>
</tr>
<tr>
<td>2030</td>
<td>1.411</td>
</tr>
</tbody>
</table>

a. Find a logistic function to model the data.

b. Based on the model, at what population does the 2000 census predict Maine’s growth to level off?

c. Discuss the effectiveness of the model to predict the population as time increases significantly beyond the domain of the data.

SOLUTION:

a. Enter the data into the calculator and then select Logistic.

b. The numerator of the logistic function is 1.43, so, according to the function, Maine’s growth will level off at about 1.43 million.

c. Sample answer: The model only remains effective as long as the birth and death rates remain consistent and the available resources do not change.
3-5 Modeling with Nonlinear Regression

16. SCUBA DIVING  Scuba divers search for dive locations with good visibility, which can be affected by the murkiness of the water and the penetration of surface light. The table shows the percent of surface light reaching a diver at different depths as the diver descends.

<table>
<thead>
<tr>
<th>Depth (ft)</th>
<th>Light (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>89.2</td>
</tr>
<tr>
<td>30</td>
<td>79.6</td>
</tr>
<tr>
<td>45</td>
<td>71.0</td>
</tr>
<tr>
<td>60</td>
<td>63.3</td>
</tr>
<tr>
<td>75</td>
<td>56.5</td>
</tr>
<tr>
<td>90</td>
<td>50.4</td>
</tr>
<tr>
<td>105</td>
<td>44.9</td>
</tr>
<tr>
<td>120</td>
<td>40.1</td>
</tr>
</tbody>
</table>

a. Use the regression features on a calculator to determine the regression equation that best relates the data.
b. Use the graph of your regression equation to approximate the percent of surface light that reaches the diver at a depth of 83 feet.

*SOLUTION:*
a. The quadratic, cubic, quartic, and exponential regressions have very similar values of $r$. Each of these appear to relate the data.

The exponential regression is approximately $y = 100(0.99)^x$.
b. Use the graph to find the value of $y$ at $x = 83$.

About 53% of surface light will reach the diver at a depth of 83 feet.

17. EELS  The table shows the average length of female king snake eels at various ages.

<table>
<thead>
<tr>
<th>Age (yr)</th>
<th>Length (in.)</th>
<th>Age (yr)</th>
<th>Length (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>8</td>
<td>14</td>
<td>17</td>
</tr>
<tr>
<td>6</td>
<td>11</td>
<td>16</td>
<td>18</td>
</tr>
<tr>
<td>8</td>
<td>13</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
<td>20</td>
<td>19</td>
</tr>
<tr>
<td>12</td>
<td>16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Use the regression features on a calculator to determine if a logarithmic regression is better than a logistic regression. Explain.
b. Use the graph of your regression equation to approximate the length of an eel at 19 years.

*SOLUTION:*

---

[Sample answer: once the car reaches 60 miles per hour, it has stopped accelerating.]
3-5 Modeling with Nonlinear Regression

a. Calculate the logarithmic regression.

\[
\text{LogReg}
\begin{array}{l}
y = a + b \ln x \\
a = -1.19551684 \\
b = 6.825256302 \\
r^2 = 0.9924965635 \\
r = 0.9962412175
\end{array}
\]

The rounded logarithmic regression equation is \( y = -1.19 + 6.83 \ln x \). Calculate the logistic regression.

\[
\text{Logistic}
\begin{array}{l}
y = \frac{c}{1 + ae^{-bx}} \\
a = 3.561743775 \\
b = 2.53939581 \\
c = 18.9940731
\end{array}
\]

The rounded logistic regression equation is \( y = \frac{18.99}{1 + 3.56e^{0.25x}} \). Since we cannot calculate \( r \) for the logistic regression, we need to use the graphs of the regressions and compare them to the scatter plot of the data.

Logarithmic graph:

![Logarithmic graph]

Logistic graph:

![Logistic graph]

Both the logarithmic and logistic regressions can be used to model the data, but logistic is a better model because the king snake eel does not grow indefinitely long.

b. Use the graphs to find the value of \( y \) at \( x = 19 \) for each regression.

Logarithmic: 18.91 in.

Logistic: 18.47 in.
3-5 Modeling with Nonlinear Regression

For Exercises 18-21, complete each step.

a. Linearize the data according to the given model.
b. Graph the linearized data, and find the linear regression equation.
c. Use the linear model to find a model for the original data. Check its accuracy by graphing.
3-5 Modeling with Nonlinear Regression

18. exponential

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>1</td>
<td>32</td>
</tr>
<tr>
<td>2</td>
<td>91</td>
</tr>
<tr>
<td>3</td>
<td>268</td>
</tr>
<tr>
<td>4</td>
<td>808</td>
</tr>
<tr>
<td>5</td>
<td>2400</td>
</tr>
<tr>
<td>6</td>
<td>7000</td>
</tr>
<tr>
<td>7</td>
<td>22,000</td>
</tr>
</tbody>
</table>

**SOLUTION:**

**a.** For an exponential model, replace \((x, y)\) with \((x, \ln y)\).

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.40</td>
</tr>
<tr>
<td>1</td>
<td>3.47</td>
</tr>
<tr>
<td>2</td>
<td>4.51</td>
</tr>
<tr>
<td>3</td>
<td>5.59</td>
</tr>
<tr>
<td>4</td>
<td>6.69</td>
</tr>
<tr>
<td>5</td>
<td>7.78</td>
</tr>
<tr>
<td>6</td>
<td>8.85</td>
</tr>
<tr>
<td>7</td>
<td>10.0</td>
</tr>
</tbody>
</table>

**b.** Calculate the linear regression.

\[
\begin{align*}
\text{LinReg} & \quad y = ax + b \\
 a & = 1.084126589 \\
b & = 2.3675232802 \\
r & = 0.9999320593 \\
r & = 0.9999510284
\end{align*}
\]

The rounded regression equation is \(\hat{y} = 1.08x + 2.37\). Graph the linearized data.

**c.** Replace \(\hat{y}\) with \(\ln y\) and solve for \(y\).

\[
\begin{align*}
\hat{y} & = 1.08x + 2.37 \\
\ln y & = 1.08x + 2.37 \\
e^{\ln y} & = e^{1.08x + 2.37} \\
y & = e^{1.08x} \cdot e^{2.37} \\
y & \approx 10.7e^{1.08x}
\end{align*}
\]

Graph the model and the original data.
3-5 Modeling with Nonlinear Regression

19. quadratic

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0</td>
</tr>
<tr>
<td>1</td>
<td>6.6</td>
</tr>
<tr>
<td>2</td>
<td>17.0</td>
</tr>
<tr>
<td>3</td>
<td>32.2</td>
</tr>
<tr>
<td>4</td>
<td>52.2</td>
</tr>
<tr>
<td>5</td>
<td>77.0</td>
</tr>
<tr>
<td>6</td>
<td>106.6</td>
</tr>
<tr>
<td>7</td>
<td>141.0</td>
</tr>
</tbody>
</table>

**SOLUTION:**

a. For a quadratic model, replace \((x, y)\) with \((x, \sqrt{y})\).

<table>
<thead>
<tr>
<th>x</th>
<th>(\sqrt{y})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2.57</td>
</tr>
<tr>
<td>2</td>
<td>4.12</td>
</tr>
<tr>
<td>3</td>
<td>5.67</td>
</tr>
<tr>
<td>4</td>
<td>7.22</td>
</tr>
<tr>
<td>5</td>
<td>8.77</td>
</tr>
<tr>
<td>6</td>
<td>10.32</td>
</tr>
<tr>
<td>7</td>
<td>11.87</td>
</tr>
</tbody>
</table>

b. Calculate the linear regression.

![Linear regression graph]

The rounded regression equation is \(\hat{y} = 1.55x + 1.01\). Graph the linearized data.

![Linearized data graph]

c. Replace \(\hat{y}\) with \(\sqrt{y}\) and solve for \(y\).

\[
\begin{align*}
\hat{y} &= 1.55x + 1.01 \\
\sqrt{y} &= 1.55x + 1.01 \\
y &= (1.55x + 1.01)^2 \\
y &\approx 2.4x^2 + 3.1x + 1
\end{align*}
\]

Graph the model and the original data.
3-5 Modeling with Nonlinear Regression

20. logarithmic

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>80.0</td>
</tr>
<tr>
<td>4</td>
<td>83.5</td>
</tr>
<tr>
<td>6</td>
<td>85.5</td>
</tr>
<tr>
<td>8</td>
<td>87.0</td>
</tr>
<tr>
<td>10</td>
<td>88.1</td>
</tr>
<tr>
<td>12</td>
<td>89.0</td>
</tr>
<tr>
<td>14</td>
<td>90.0</td>
</tr>
<tr>
<td>16</td>
<td>90.5</td>
</tr>
</tbody>
</table>

SOLUTION:

a. For a logarithmic model, replace \((x, y)\) with \((\ln x, y)\).

\[
\begin{array}{c|c|c|c|c|c|c|c|c}
\ln x & 0.69 & 1.39 & 1.79 & 2.68 & 2.30 & 2.48 & 2.64 & 2.77 \\
y     & 80 & 83.5 & 85.5 & 87 & 88.1 & 89 & 90 & 90.5 \\
\end{array}
\]

b. Calculate the linear regression.

\[
y = ax + b
\]

\[
\begin{array}{l}
\hat{a} = 5.075180465 \\
b = 76.454619728 \\
r = 0.9998155102
\end{array}
\]

The rounded regression equation is \(y = 5.1\hat{x} + 76.5\).

Graph the linearized data.

![Graph of linearized data]

\([0, 3] \text{ scl: 0.5 by [0, 100] scl: 10]\)

c. Replace \(\hat{x}\) with \(\ln x\) and simplify.

\[
y = 5.1\ln x + 76.5
\]

Graph the model and the original data.

![Graph of model and original data]

\([0, 20] \text{ scl: 2 by [0, 100] scl: 10]\)

21. power
For Exercises 1 – 3, complete each step.

a. Find an exponential function to model the data.

b. Find the value of each... time from \(0 \leq t \leq 2\).

c. Replace \( \hat{x} \) with \( \ln x \) and replace \( \hat{y} \) with \( \ln y \) and solve for \( y \).

\[
\frac{y}{x^{1.99}} = 1.62 \\
\frac{y}{x^{1.99}} = e^{1.62} \\
y = e^{1.62}x^{1.99} \\
y \approx 5.05x^{1.99}
\]

Graph the model and the original data.

---

**SOLUTION:**

a. For a power model, replace \((x, y)\) with \((\ln x, \ln y)\).

<table>
<thead>
<tr>
<th>(\ln x)</th>
<th>0</th>
<th>0.69</th>
<th>1.10</th>
<th>1.39</th>
<th>1.61</th>
<th>1.79</th>
<th>1.95</th>
<th>2.08</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\ln y)</td>
<td>1.61</td>
<td>3.04</td>
<td>3.78</td>
<td>4.37</td>
<td>4.79</td>
<td>5.19</td>
<td>5.52</td>
<td>5.77</td>
</tr>
</tbody>
</table>

b. Calculate the linear regression.

The rounded regression equation is \(\hat{y} = 1.99\hat{x} + 1.62\).

Graph the linearized data.

\([0, 2.5] \text{ sel: } 0.25 \times [0, 10] \text{ sel: } 1\)

c. Replace \(\hat{x}\) with \(\ln x\) and replace \(\hat{y}\) with \(\ln y\) and solve for \(y\).

\[
\begin{align*}
\hat{y} &= 1.99\hat{x} + 1.62 \\
\ln y &= 1.99\ln x + 1.62 \\
\ln y - 1.99\ln x &= 1.62 \\
\ln y - \ln x^{1.99} &= 1.62 \\
\ln \frac{y}{x^{1.99}} &= 1.62 \\
\frac{y}{x^{1.99}} &= e^{1.62} \\
y &= e^{1.62}x^{1.99} \\
y &\approx 5.05x^{1.99}
\end{align*}
\]
22. TORNADOES A tornado with a greater wind speed near the center of its funnel can travel greater distances. The table shows the wind speeds near the centers of tornadoes that have traveled various distances.

<table>
<thead>
<tr>
<th>Distance (mi)</th>
<th>Wind Speed (mph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>37</td>
</tr>
<tr>
<td>0.75</td>
<td>53</td>
</tr>
<tr>
<td>1.00</td>
<td>65</td>
</tr>
<tr>
<td>1.25</td>
<td>74</td>
</tr>
<tr>
<td>1.50</td>
<td>81</td>
</tr>
<tr>
<td>1.75</td>
<td>80</td>
</tr>
<tr>
<td>2.00</td>
<td>93</td>
</tr>
<tr>
<td>2.25</td>
<td>98</td>
</tr>
<tr>
<td>2.50</td>
<td>102</td>
</tr>
<tr>
<td>2.75</td>
<td>106</td>
</tr>
</tbody>
</table>

a. Linearize the data assuming a logarithmic model.
b. Graph the linearized data, and find the linear regression equation.
c. Use the linear model to find a model for the original data, and approximate the wind speed of a funnel that has traveled 3.7 miles.

*SOLUTION:*

a. For a logarithmic model, replace \((x, y)\) with \((\ln x, y)\).

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c|c|c|c|c|c}
x & -1.69 & -0.29 & 0 & 0.22 & 0.41 & 0.56 & 0.69 & 0.81 & 0.92 & 1.01 \\
\ln x & \text{y} \\
& 37 & 53 & 65 & 74 & 81 & 88 & 93 & 98 & 102 & 106
\end{array}
\]

b. Calculate the linear regression.

\[
\begin{align*}
\text{LinReg} \\
y = ax + b \\
a = \frac{40.5752104}{64.933801721} = 0.6426 \\
b = 64.933801721 \\
r = 0.9999928354
\end{align*}
\]

The rounded regression equation is \(y = 40.6\hat{x} + 64.9\).

Graph the linearized data.
3-5 Modeling with Nonlinear Regression

c. Replace \( \hat{y} \) with \( \ln x \) and simplify.
\[
y = 40.6\hat{y} + 64.9
\]
\[
y = 40.6 \ln x + 64.9
\]
Substitute 3.7 for \( x \).
\[
y = 40.6 \ln 3.7 + 64.9
\]
\[
y \approx 118
\]
3-5 Modeling with Nonlinear Regression

23. **HOUSING** The table shows the appreciation in the value of a house every 3 years since the house was purchased.

<table>
<thead>
<tr>
<th>Years</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value ($)</td>
<td>78,000</td>
<td>81,576</td>
<td>85,992</td>
<td>90,791</td>
<td>95,859</td>
<td>101,135</td>
</tr>
</tbody>
</table>

- **a.** Linearize the data assuming an exponential model.
- **b.** Graph the linearized data, and find the linear regression equation.
- **c.** Use the linear model to find a model for the original data, and approximate the value of the house 24 years after it is purchased.

**SOLUTION:**

- **a.** For an exponential model, replace \((x, y)\) with \((x, \ln y)\).

<table>
<thead>
<tr>
<th>(x)</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\ln y)</td>
<td>11.26</td>
<td>11.31</td>
<td>11.36</td>
<td>11.42</td>
<td>11.47</td>
<td>11.52</td>
</tr>
</tbody>
</table>

- **b.** Calculate the linear regression.

```
LinReg
\hat{y} = a + bx
a = 0.174393317
b = 11.26
r = 0.995031223
r^2 = 0.9915379
```

The rounded regression equation is \(\hat{y} = 0.0175x + 11.26\).

Graph the linearized data.

- **c.** Replace \(\hat{y}\) with \(\ln y\) and solve for \(y\). Then substitute 24 for \(x\).

\(\hat{y} = 0.0175x + 11.26\)

\(\ln y = 0.0175x + 11.26\)

\(y = e^{0.0175x + 11.26}\)

\(y = e^{0.0175x} \cdot e^{11.26}\)

\(y \approx 77,653e^{0.0175x}\)

\(y \approx 77,653e^{0.0175(24)}\)

\(y \approx 118,185\)

24. **COOKING** Cooking times, temperatures, and recipes are often different at high altitudes than at sea level. This is due to the difference in atmospheric pressure, which causes boiling points to be lower at higher altitudes. The table shows the boiling point of water at different elevations above sea level.
3-5 Modeling with Nonlinear Regression

<table>
<thead>
<tr>
<th>Elevation (m)</th>
<th>Boiling Point (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>1000</td>
<td>99.29</td>
</tr>
<tr>
<td>2000</td>
<td>98.81</td>
</tr>
<tr>
<td>3000</td>
<td>98.43</td>
</tr>
<tr>
<td>4000</td>
<td>98.10</td>
</tr>
<tr>
<td>5000</td>
<td>97.80</td>
</tr>
<tr>
<td>6000</td>
<td>97.53</td>
</tr>
<tr>
<td>7000</td>
<td>97.28</td>
</tr>
<tr>
<td>8000</td>
<td>97.05</td>
</tr>
<tr>
<td>9000</td>
<td>96.83</td>
</tr>
<tr>
<td>10,000</td>
<td>96.62</td>
</tr>
</tbody>
</table>

a. Make a scatter plot of the data.
b. Linearize the data for exponential, power, and logarithmic models.
c. Graph the linearized data, and determine which model best represents the data.
d. Write an equation to model the data based on your analysis of the linearizations.

*SOLUION:*

a. For an exponential model, replace \((x, y)\) with \((x, \ln y)\). For a power model, replace \((x, y)\) with \((\ln x, \ln y)\). For a logarithmic model, replace \((x, y)\) with \((\ln x, y)\).

<table>
<thead>
<tr>
<th>Elevation (m)</th>
<th>Exponential</th>
<th>Power</th>
<th>Logarithmic</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.605</td>
<td>undef</td>
<td>4.605</td>
</tr>
<tr>
<td>1000</td>
<td>4.598</td>
<td>6.91</td>
<td>4.598</td>
</tr>
<tr>
<td>2000</td>
<td>4.593</td>
<td>7.60</td>
<td>4.593</td>
</tr>
<tr>
<td>3000</td>
<td>4.589</td>
<td>8.01</td>
<td>4.589</td>
</tr>
<tr>
<td>4000</td>
<td>4.586</td>
<td>8.29</td>
<td>4.586</td>
</tr>
<tr>
<td>5000</td>
<td>4.583</td>
<td>8.52</td>
<td>4.593</td>
</tr>
<tr>
<td>6000</td>
<td>4.580</td>
<td>8.70</td>
<td>4.580</td>
</tr>
<tr>
<td>7000</td>
<td>4.578</td>
<td>8.85</td>
<td>4.578</td>
</tr>
<tr>
<td>8000</td>
<td>4.575</td>
<td>8.99</td>
<td>4.575</td>
</tr>
<tr>
<td>9000</td>
<td>4.573</td>
<td>9.10</td>
<td>4.573</td>
</tr>
<tr>
<td>10,000</td>
<td>4.571</td>
<td>9.21</td>
<td>4.571</td>
</tr>
</tbody>
</table>

b. Calculate each regression.

Exponential:

```
LinReg
y=ax+b
a=3.2320315-6
b=4.60883232
r^2=0.9671434496
r=-0.9834345167
```

Power:

```
LinReg
y=ax+b
a=-0.0033317743
b=4.61916992
r^2=0.6471108957
r=-0.8044319358
```
3-5 Modeling with Nonlinear Regression

Graph each regression.
Exponential:

Power:

Logarithmic:

The best representation is given by the exponential model because the linearized graph resembles a line the most. Also, the absolute value of the correlation coefficient is the closest to 1.

c. The rounded linearized exponential regression equation is \( \hat{y} = -0.0000032x + 4.6 \). Replace \( \hat{y} \) with \( \ln y \) and solve for \( y \).

\[
\ln y = -0.0000032x + 4.6
\]

\[
y = e^{-0.0000032x+4.6}
\]

\[
y = e^{-0.0000032} \cdot e^{4.6}
\]

\[
y \approx 99.48 e^{-0.0000032x}
\]
3-5 Modeling with Nonlinear Regression

Determine the model most appropriate for each scatter plot. Explain your reasoning.

25.

SOLUTION:
For a logarithmic or exponential model, rate of change of $y$ would either increase or decrease dramatically as $x$ increased. In this graph, however, the graph appears to increase at a constant percent change, so the graph can be represented by a power function.

26.

SOLUTION:
The graph increases at a decreasing rate and then levels off. It appears that the $y$-values will never exceed a certain point. This represents a logistic function.

Linearize the data in each table. Then determine the most appropriate model.

27.

SOLUTION:
For an exponential model, replace $(x, y)$ with $(x, \ln y)$. For a power model, replace $(x, y)$ with $(\ln x, \ln y)$. For a logarithmic model, replace $(x, y)$ with $(\ln x, y)$. Linearize the data according to each model and sketch the graphs.
3-5 Modeling with Nonlinear Regression

<table>
<thead>
<tr>
<th></th>
<th>Exponential</th>
<th>Power</th>
<th>Logarithmic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>( \ln y )</td>
<td>( \ln x )</td>
<td>( \ln x )</td>
</tr>
<tr>
<td>2</td>
<td>0.92</td>
<td>0.69</td>
<td>0.69</td>
</tr>
<tr>
<td>4</td>
<td>1.99</td>
<td>1.39</td>
<td>1.39</td>
</tr>
<tr>
<td>6</td>
<td>2.62</td>
<td>1.79</td>
<td>1.79</td>
</tr>
<tr>
<td>8</td>
<td>3.06</td>
<td>2.08</td>
<td>2.08</td>
</tr>
<tr>
<td>10</td>
<td>3.41</td>
<td>2.30</td>
<td>2.30</td>
</tr>
<tr>
<td>12</td>
<td>3.69</td>
<td>2.48</td>
<td>2.48</td>
</tr>
<tr>
<td>14</td>
<td>3.93</td>
<td>2.64</td>
<td>2.64</td>
</tr>
<tr>
<td>16</td>
<td>4.14</td>
<td>2.77</td>
<td>2.77</td>
</tr>
</tbody>
</table>

Exponential:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>20.0</td>
</tr>
<tr>
<td>2.0</td>
<td>2.05</td>
</tr>
<tr>
<td>4.0</td>
<td>6.32</td>
</tr>
<tr>
<td>6.0</td>
<td>19.0</td>
</tr>
<tr>
<td>8.0</td>
<td>83.3</td>
</tr>
</tbody>
</table>

Power:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.3</td>
</tr>
<tr>
<td>2.0</td>
<td>0.5</td>
</tr>
<tr>
<td>3.0</td>
<td>0.7</td>
</tr>
<tr>
<td>4.0</td>
<td>0.9</td>
</tr>
<tr>
<td>5.0</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Logarithmic:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.3</td>
</tr>
<tr>
<td>2.0</td>
<td>0.5</td>
</tr>
<tr>
<td>3.0</td>
<td>0.7</td>
</tr>
<tr>
<td>4.0</td>
<td>0.9</td>
</tr>
<tr>
<td>5.0</td>
<td>1.1</td>
</tr>
</tbody>
</table>

The graph of the linearized power regression most closely resembles a line.

28.

**SOLUTION:**

...
3-5 Modeling with Nonlinear Regression

For an exponential model, replace \((x, y)\) with \((x, \ln y)\). For a power model, replace \((x, y)\) with \((\ln x, \ln y)\). For a logarithmic model, replace \((x, y)\) with \((\ln x, y)\). Linearize the data according to each model and sketch the graphs.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(\ln y)</th>
<th>(\ln x)</th>
<th>(\ln x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.79</td>
<td>0.00</td>
<td>1.79</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>3.36</td>
<td>0.69</td>
<td>3.37</td>
<td>29</td>
</tr>
<tr>
<td>3</td>
<td>3.74</td>
<td>1.10</td>
<td>3.74</td>
<td>42</td>
</tr>
<tr>
<td>4</td>
<td>3.95</td>
<td>1.39</td>
<td>3.95</td>
<td>52</td>
</tr>
<tr>
<td>5</td>
<td>4.08</td>
<td>1.61</td>
<td>4.08</td>
<td>59</td>
</tr>
<tr>
<td>6</td>
<td>4.18</td>
<td>1.79</td>
<td>4.17</td>
<td>65</td>
</tr>
<tr>
<td>7</td>
<td>4.25</td>
<td>1.95</td>
<td>4.25</td>
<td>70</td>
</tr>
<tr>
<td>8</td>
<td>4.31</td>
<td>2.08</td>
<td>4.32</td>
<td>75</td>
</tr>
</tbody>
</table>

Exponential:

Power:

Logarithmic:

The graph of the linearized logarithmic regression most closely resembles a line.
3-5 Modeling with Nonlinear Regression

**SOLUTION:**

For an exponential model, replace \((x, y)\) with \((x, \ln y)\). For a power model, replace \((x, y)\) with \((\ln x, \ln y)\). For a logarithmic model, replace \((x, y)\) with \((\ln x, y)\). Linearize the data according to each model and sketch the graphs.

<table>
<thead>
<tr>
<th>Exponential</th>
<th>Power</th>
<th>Logarithmic</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x)</td>
<td>(\ln x)</td>
<td>(\ln x)</td>
</tr>
<tr>
<td>1</td>
<td>3.63</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2.83</td>
<td>0.69</td>
</tr>
<tr>
<td>3</td>
<td>2.04</td>
<td>1.10</td>
</tr>
<tr>
<td>4</td>
<td>1.22</td>
<td>1.39</td>
</tr>
<tr>
<td>5</td>
<td>0.47</td>
<td>1.61</td>
</tr>
<tr>
<td>6</td>
<td>-0.36</td>
<td>1.79</td>
</tr>
<tr>
<td>7</td>
<td>-1.20</td>
<td>1.95</td>
</tr>
<tr>
<td>8</td>
<td>-2.30</td>
<td>2.08</td>
</tr>
</tbody>
</table>

Exponential:

![Graph of Exponential Linearization](image)

Power:

![Graph of Power Linearization](image)

Logarithmic:

![Graph of Logarithmic Linearization](image)

The graph of the linearized exponential regression most closely resembles a line.

30. **FISH** Several ichthyologists are studying the smallmouth bass population in a lake. The table shows the smallmouth bass population of the lake over time.

<table>
<thead>
<tr>
<th>Year</th>
<th>Bass</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>673</td>
</tr>
<tr>
<td>2002</td>
<td>891</td>
</tr>
<tr>
<td>2003</td>
<td>1453</td>
</tr>
<tr>
<td>2004</td>
<td>1889</td>
</tr>
<tr>
<td>2005</td>
<td>2542</td>
</tr>
<tr>
<td>2006</td>
<td>2967</td>
</tr>
<tr>
<td>2007</td>
<td>3018</td>
</tr>
<tr>
<td>2008</td>
<td>3011</td>
</tr>
</tbody>
</table>

**a.** Make a scatter plot of the data.

**b.** Determine the most appropriate model for the data. Explain your reasoning.
3-5 Modeling with Nonlinear Regression

c. Find a function to model the data.
d. Use the function to predict the smallmouth bass population in 2012.
e. Discuss the effectiveness of the model to predict the population of the bass as time increases significantly beyond the domain of the data.

**SOLUTION:**
a. Make a scatter plot of the data. Let \( x \) be the number of years after 2000.

\[
\begin{array}{c}
[0, 10] \text{ scl: 1 by [0, 3500] scl: 500}
\end{array}
\]

The shape of the graph appears to be logistic. Find the logistic regression equation and sketch the graph with the scatter plot.

\[
\begin{array}{c}
[0, 10] \text{ scl: 1 by [0, 3500] scl: 500}
\end{array}
\]

The shape may also be logarithmic. Sketch this regression as well.

\[
\begin{array}{c}
[0, 10] \text{ scl: 1 by [0, 3500] scl: 500}
\end{array}
\]

The graph more closely resembles a logistic model.

b. 

\[
y = \frac{3239}{1 + 9.84e^{-0.696x}}
\]

c. Substitute 12 for \( x \) in the full logistic regression equation.
3-5 Modeling with Nonlinear Regression

The population will be about 3231.

d. Sample answer: While this model provides a pretty effective representation and can fairly accurately predict the bass population in the future, the model cannot take into consideration any changes to the lake caused by the atmosphere or by humans. For example, a nearby company could dump waste into the lake, which would affect the increasing bass population.

31. REASONING Why are logarithmic regressions invalid when the domain is 0?

SOLUTION:
Sample answer: Logarithmic regression is invalid when the domain is 0 because logarithmic functions are undefined at that value and regression does not work with undefined values.

32. CHALLENGE Show that \(y = ab^x\) can be converted to \(y = ae^{kx}\).

SOLUTION:

\[
\begin{align*}
ab^x &= ae^{kx} \\
b^x &= e^{kx} \\
\ln b^x &= \ln e^{kx} \\
x \ln b &= kx \\
\ln b &= k \\
\text{Therefore,} \quad y &= ae^{kx} \\
y &= ae^{x \ln b} \\
y &= ae^{\ln b^x} \\
y &= ab^x
\end{align*}
\]

33. REASONING Can the graph of a logistic function ever have any intercepts? Explain your reasoning.

SOLUTION:
Sample answer: Yes; a logistic function has a \(y\)-intercept at \(y = \frac{c}{1 + ae}\), but cannot have an \(x\)-intercept because a constant can never be zero.
3-5 Modeling with Nonlinear Regression

PROOF Use algebra to verify that data modeled by each type of function can be linearized, or expressed as a function \( y = mx + b \) for some values \( m \) and \( b \), by replacing \((x, y)\) with the indicated coordinates.

34. exponential, \((x, \ln y)\)

**SOLUTION:**
Substitute \( \ln y \) for \( y \) in the general exponential equation and simplify to determine if the linearized function appears to be linear.

\[
y = ab^x
\]
\[
\ln y = \ln ab^x
\]
\[
\ln y = \ln a + \ln b^x
\]
\[
\ln y = \ln a + x \ln b
\]
\[
\ln y = (\ln b)x + \ln a
\]
Since \( \ln a \) and \( \ln b \) are constants, this equation is linear when \( y = \ln y \).

35. power, \((\ln x, \ln y)\)

**SOLUTION:**
Substitute \( \ln y \) for \( y \) and \( \ln x \) for \( x \) in the general power equation and simplify to determine if the linearized function appears to be linear.

\[
y = ax^b
\]
\[
\ln y = \ln ax^b
\]
\[
\ln y = \ln a + \ln x^b
\]
\[
\ln y = b \ln x + \ln a
\]
Since \( \ln a \) and \( b \) are constants, this equation is linear when \( y = \ln y \) and \( x = \ln x \).

36. **REASONING** How is the graph of \( g(x) = \frac{5}{1 + ex} + a \) related to the graph of \( f(x) = \frac{5}{1 + ex} \)? Explain.

**SOLUTION:**
\( G(x) \) will be a vertical shift of the graph \( f(x) \). The graph will be shifted \( a \) units up if \( a \) is positive, and \( a \) units down if \( a \) is negative.

37. **Writing in Math** Explain how the parameters of an exponential or logarithmic model relate to the data set or situation being modeled.

**SOLUTION:**
Sample answer: Data sets modeled by exponential regression have a restricted range. Data sets modeled by logarithmic regression have a restricted domain.
3-5 Modeling with Nonlinear Regression

Solve each equation.

38. $3^{4x} = 3^{3-x}$

**SOLUTION:**

$3^{4x} = 3^{3-x}$

$4x = 3 - x$

$5x = 3$

$x = \frac{3}{5}$

39. $3^{5x} \cdot 81^{1-x} = 9^{x-3}$

**SOLUTION:**

$3^{5x} \cdot 81^{1-x} = 9^{x-3}$

$3^{5x} \cdot (3^4)^{1-x} = (3^2)^{x-3}$

$3^{5x} \cdot 3^{4-4x} = 3^{2x-6}$

$3^{5x+4-4x} = 3^{2x-6}$

$3^x = 3^{2x-6}$

$x + 4 = 2x - 6$

$10 = x$

40. $49^x = 7^{x^2-15}$

**SOLUTION:**

$49^x = 7^{x^2-15}$

$(7^2)^x = 7^{x^2-15}$

$7^{2x} = 7^{x^2-15}$

$2x = x^2 - 15$

$0 = x^2 - 2x - 15$

$0 = (x-5)(x+3)$

41. $\log_5 (4x - 1) = \log_5 (3x + 2)$

**SOLUTION:**

$\log_5 (4x - 1) = \log_5 (3x + 2)$

$4x - 1 = 3x + 2$

$x = 3$
3-5 Modeling with Nonlinear Regression

42. \( \log_{10} z + \log_{10} (z + 3) = 1 \)

**SOLUTION:**

\[
\log_{10} z + \log_{10} (z + 3) = 1 \\
\log_{10} [z(z + 3)] = 1 \\
10^{\log_{10}[z(z+3)]} = 10^1 \\
z(z + 3) = 10 \\
z^2 + 3z = 10 \\
z^2 + 3z - 10 = 0 \\
(z + 5)(z - 2) = 0 \\
z = -5 \text{ or } 2
\]

The logarithm of a negative number provides no real solution, so 2 is the only solution.

43. \( \log_{6} (a^2 + 2) + \log_{6} 2 = 2 \)

**SOLUTION:**

\[
\log_{6} (a^2 + 2) + \log_{6} 2 = 2 \\
\log_{6} (2a^2 + 4) = 2 \\
6^{\log_{6}(2a^2+4)} = 6^2 \\
2a^2 + 4 = 36 \\
2a^2 = 32 \\
a^2 = 16 \\
a = \pm 4
\]
3-5 Modeling with Nonlinear Regression

44. ENERGY The energy \( E \), in kilocalories per gram molecule, needed to transport a substance from the outside to the inside of a living cell is given by \( E = 1.4(\log_{10} C_2 - \log_{10} C_1) \), where \( C_1 \) is the concentration of the substance outside the cell and \( C_2 \) is the concentration inside the cell.

a. Express the value of \( E \) as one logarithm.

b. Suppose the concentration of a substance inside the cell is twice the concentration outside the cell. How much energy is needed to transport the substance on the outside of the cell to the inside? (Use \( \log_{10} 2 \approx 0.3010 \).

c. Suppose the concentration of a substance inside the cell is four times the concentration outside the cell. How much energy is needed to transport the substance from the outside of the cell to the inside?

\[ \text{SOLUTION:} \]

\[ \begin{align*}
\text{a.} & \quad E = 1.4(\log_{10} C_2 - \log_{10} C_1) \\
& = 1.4 \log_{10} \frac{C_2}{C_1} \\
\text{b.} & \quad C_2 \text{ is twice } C_1, \text{ so } C_2 = 2C_1. \\
& \quad E = 1.4 \log_{10} \frac{2C_1}{C_1} \\
& \quad = 1.4 \log_{10} 2 \\
& \quad \approx 0.4214 \\
\text{c.} & \quad C_2 \text{ is four times } C_1, \text{ so } C_2 = 4C_1. \\
& \quad E = 1.4 \log_{10} \frac{4C_1}{C_1} \\
& \quad = 1.4 \log_{10} 4 \\
& \quad \approx 0.8429
\]
3-5 Modeling with Nonlinear Regression

45. FINANCIAL LITERACY  In 2003, Maya inherited $1,000,000 from her grandmother. She invested all of the money; and by 2015, the amount will grow to $1,678,000.

a. Write an exponential function that could be used to model the amount of money $y$. Write the function in terms of $x$, the number of years since 2003.
b. Assume that the amount of money continues to grow at the same rate. Estimate the amount of money in 2025. Is this estimate reasonable? Explain your reasoning.

**SOLUTION:**

a. The general exponential function is $N = N_0(1 + r)^t$. The time $t$ is $2015 - 2003$ or 12. Solve for the rate $r$.

\[
N = N_0(1 + r)^t \\
1,678,000 = 1,000,000(1 + r)^{12} \\
1.678 = (1 + r)^{12} \\
\ln 1.678 = \ln(1 + r)^{12} \\
\ln 1.678 = 12 \ln(1 + r) \\
\frac{\ln 1.678}{12} = \ln(1 + r) \\
e^{12 \ln 1.678} = e^{\ln(1 + r)} \\
e^{12} = 1 + r \\
e^{12} - 1 = r \\
0.044 \approx r
\]

Substitute $x$ for $t$, 0.044 for $r$, and 1,000,000 for $N_0$ in the original equation.

\[
N = N_0(1 + 0.044)^t \\
N = N_0(1.044)^t \\
N = 1,000,000(1.044)^t
\]
b. The time $t = 2025 - 2003$ or 22.

\[
N = 1,000,000(1.044)^{22} \\
= 2,578,760.27
\]
3-5 Modeling with Nonlinear Regression

Simplify.
46. \((-2i)(-6i)(4i)\)

**SOLUTION:**
\[
(-2i)(-6i)(4i) = 48i^3
= 48i^2 \cdot i
= 48(-1) \cdot i
= -48i
\]

47. \(3i(-5i)^2\)

**SOLUTION:**
\[
(3i)(-5i)^2 = 3(25)i^3
= 75i^2 \cdot i
= 75(-1) \cdot i
= -75i
\]

48. \(i^{13}\)

**SOLUTION:**
\[
i^{13} = i^{12}i
= (i^4)^3i
= 1^3 \cdot i
= i
= -75i
\]

49. \((1 - 4i)(2 + i)\)

**SOLUTION:**
\[
(1 - 4i)(2 + i) = 2 + i - 8i - 4i^2
= 2 - 7i - 4(-1)
= 2 - 7i + 4
= 6 - 7i
\]
3-5 Modeling with Nonlinear Regression

50. \( \frac{4i}{3+i} \)

**SOLUTION:**

\[
\frac{4i}{3+i} = \frac{4i}{3+i} \cdot \frac{3-i}{3-i} = \frac{12i - 4i^2}{9 - i^2} = \frac{12i - 4(-1)}{9 - (-1)} = \frac{12i + 4}{10} = \frac{6i + 2}{5} = \frac{6}{5} i + \frac{2}{5}
\]

51. \( \frac{4}{5+3i} \)

**SOLUTION:**

\[
\frac{4}{5+3i} = \frac{4}{5+3i} \cdot \frac{5-3i}{5-3i} = \frac{20 - 12i}{25 - 9i^2} = \frac{20 - 12i}{25 - 9(-1)} = \frac{20 - 12i}{25 + 9} = \frac{20 - 12i}{34} = \frac{10 - 6i}{17} = \frac{10}{17} - \frac{6}{17} i
\]
3-5 Modeling with Nonlinear Regression

52. SAT/ACT  A recent study showed that the number of Australian homes with a computer doubles every 8 months. Assuming that the number is increasing continuously, at approximately what monthly rate must the number of Australian computer owners be increasing for this to be true?

A 6.8%
B 8.66%
C 12.5%
D 8.0%
E 2%

**SOLUTION:**

Use the formula for exponential regression. The value of time \( t \) is 8. Since we are looking for the initial amount to double, \( N = 2N_0 \).

\[
\begin{align*}
N &= N_0e^{kt} \\
2N_0 &= N_0e^{kt} \\
2 &= e^{8k} \\
\ln 2 &= \ln e^{8k} \\
\ln 2 &= 8k \\
\ln 2 \cdot \frac{1}{8} &= k \\
0.0866 &\approx k
\end{align*}
\]

The correct choice is B.
3-5 Modeling with Nonlinear Regression

53. The data below gives the number of bacteria found in a certain culture. The bacteria are growing exponentially.

<table>
<thead>
<tr>
<th>Hours</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bacteria</td>
<td>5</td>
<td>8</td>
<td>15</td>
<td>26</td>
<td>48</td>
</tr>
</tbody>
</table>

Approximately how much time will it take the culture to double after hour 4?

F 1.26 hours  
G 1.35 hours  
H 1.68 hours  
J 1.76 hours

SOLUTION:

Calculate the exponential regression of the data.

![Graph showing exponential regression]

After hour 4, when the number of bacteria doubles, there will be 96 bacteria. Find the intersection of the full regression equation and the graph of \( y = 96 \).

![Graph showing intersection]

\[ 5.26 - 4 = 1.26 \text{ hours}. \]

The correct choice is F.
3-5 Modeling with Nonlinear Regression

54. **FREE RESPONSE** The speed in miles per hour at which a car travels is represented by \( v(t) = 60(1 - e^{-t^2}) \) where \( t \) is the time in seconds. Assume the car never needs to stop.

a. Graph \( v(t) \) for \( 0 \leq t \leq 10 \).

b. Describe the domain and range of \( v(t) \). Explain your reasoning.

c. What type of function is \( v(t) \)?

d. What is the end behavior of \( v(t) \)? What does this mean in the context of the situation?

e. Let \( d(t) \) represent the total distance traveled by the car. What type of function does \( d(t) \) represent as \( t \) approaches infinity? Explain.

f. Let \( a(t) \) represent the acceleration of the car. What is the end behavior of \( a(t) \)? Explain.

**SOLUTION:**

a. 

![Graph of v(t)](image)

b. \( D = \{ t | t \geq 0 \} \), \( R = \{ v | v > 0 \} \); Sample answer: Time and speed cannot be negative.

c. The graph increases then appears to reach a horizontal asymptote, so \( v(t) \) is a logistic function.

d. The graph has a horizontal asymptote at \( x = 60 \), so \( \lim_{t \to \infty} v(t) = 60 \); Sample answer: The car reaches 60 miles per hour, then remains at that speed.

e. Sample answer: A linear function; as \( t \) approaches infinity, the total distance traveled would be represented by \( d(t) = 60t \). This is due to the fact that the car has been traveling at that speed since \( t = 2 \), and as time increases, the average speed of the car is less affected by the acceleration time from \( 0 \leq t \leq 2 \).

f. Sample answer: \( \lim_{t \to \infty} a(t) = 0 \); once the car reaches 60 miles per hour, it has stopped accelerating.